

# A DETERMINATION OF THE EXTRAGALACTIC-PLANETARY FRAME TIC FROM COMPARISON OF RADIO INTERFEROMETRIC AND LUNAR LASER RANGING STATION COORDINATES

W. M. Folkner<sup>1</sup>, P. Charlot<sup>2</sup>, M. H. Finger<sup>3</sup>, X X Newhall<sup>1</sup>,  
J. G. Williams<sup>1</sup>, O. J. Sovers<sup>1</sup>, E. M. Standish<sup>1</sup>

<sup>1</sup> Jet Propulsion Laboratory, California Institute of Technology,  
4800 Oak Grove Drive, Pasadena, California 91109.

<sup>2</sup> Observatoire de Paris - CNRS/URA 1125,  
61 Avenue de l'Observatoire, 75014 Paris, France.

<sup>3</sup> Astronomy Programs, Computer Sciences Corp.,  
Marshall Space Flight Center, Alabama 35812.

## Abstract.

Very Long Baseline Interferometry (VLBI) observations of extragalactic radio sources provide the basis for defining an accurate non-rotating reference frame in terms of the angular positions of the sources. Measurements of the distance between the Earth and the moon and the inner planets provide the basis for defining an inertial planetary ephemeris reference frame. The relative orientation, or frame tic, between these two reference frames is of interest for combining Earth orientation measurements, for comparing Earth orientation results with theories referred to the mean equator and equinox, and for determining the positions of the planets with respect to the extragalactic reference frame. This work presents an indirect determination of the extragalactic-planetary frame tic from a combined reduction of VLBI and Lunar Laser Ranging (LLR) observations. For this determination, data acquired by LLR tracking stations since 1969 have been analyzed and combined with 14 years of VLBI data acquired by NASA's Deep Space Network since 1978. The frame tic derived from this joint analysis, with an accuracy of 0.003", is the most accurate determination obtained so far. This result, combined with a determination of the mean ecliptic (defined in the rotating sense), shows that the mean equinox of epoch J2000 is offset from the  $x$ -axis of the extragalactic frame adopted by the International Earth Rotation Service for astrometric and geodetic applications by 0.083:1:0.01 O" along the  $y$ -direction and by 0.0224:0.003" along the  $z$ -direction.

## 1. Introduction

Very Long Baseline Interferometry (VLBI) observations are sensitive to the relative orientation of baseline vectors between radio telescopes and directions to extragalactic radio sources. This sensitivity has been used to determine relative station locations with  $\sim 1$  cm accuracy and to monitor variations in Earth rotation with millisecond (mas) accuracy. Relative directions of radio sources are determined to mas accuracy (Sovers 1991a). In order to combine information from many different VLBI sessions and data reductions it is useful to fix the orientation of the coordinate system of the radio sources and to measure Earth orientation (precession, rotation, universal time, and polar motion) and tectonic motion with respect to this defined celestial system. Since 1988 the International Earth

Rotation Service (IERS) has maintained a celestial reference frame defined by the positions of well-observed radio sources (Arias et al., 1988). Since VLBI data are not very sensitive to the orbit of the Earth, and hence insensitive to the dynamical equinox, the rigid ascension origin of the IERS celestial system was set to agree with that defined by the historical lunar occultation of the source 3C 273 (Hazard et al. 1971). The orientation of the IERS celestial frame is nominally maintained with each new realization of the IERS celestial frame (IERS 1993).

The planetary ephemeris frame is defined by the orbits of the Earth, moon, and planets. Lunar Laser Ranging (LLR) enables high-precision studies of Earth-Moon dynamics. Through the effect of solar perturbations on the lunar orbit, LLR observations are sensitive to the direction to the Sun and to the plane defined by the Earth's orbit (the ecliptic). The lunar ephemeris can therefore be tied in orientation to the orbit of the Earth at the milliarcsecond (mas) level (Williams and Standish, 1989). In addition, LLR is sensitive to the orientation of the Earth (with respect to the lunar orbit) at the mas-level and LLR station locations can be determined with few-centimeter accuracy (Newhall et al. 1993). The orbit of Mars is tied to the Earth-moon system with  $\sim 2$  mas accuracy from six years of accurate range measurements to the Viking landers (Williams and Standish, 1989). The orbits of Venus and Mercury, with respect to the Earth-moon-Mars system, are known to  $\sim 5$  mas from radar range observations. The orbits of the outer planets are based largely on optical data and are thus known with much less accuracy. When comparing the planetary ephemeris frame to the extragalactic radio frame we are implicitly referring to the orbits of the inner planets, especially Earth and Mars.

Because both VLBI and LLR are sensitive to station locations with respect to their respective celestial reference frames, it is possible to infer the relative orientation between the extragalactic radio frame and the planetary ephemeris frame by comparing the relative orientation of VLBI and LLR terrestrial frames and the time-dependent Earth orientation transformations needed to map Earth-fixed station locations to the VLBI and LLR celestial frames. Knowledge of the intragalactic-planetary frame tie is of interest for Earth orientation studies. The reference point for theories of Earth orientation is the equinox, which is the intersection of the mean equator with the mean ecliptic of the reference epoch (e.g. J2000). LLR data are sensitive to both the equator and the ecliptic but are not as robust as VLBI for Earth orientation studies since there are fewer LLR observation stations, the LLR data of a given date occur only in a single direction on the celestial sphere, and data acquisition is limited by the phases (brightness) of the moon. Determination of the frame tie allows the routine VLBI Earth orientation results to be referred to the equinox.

Knowledge of the frame tie also specifies the orbits of the planets with respect to the extragalactic frame. This is of particular interest for the navigation of interplanetary spacecraft. Short arcs of spacecraft range and Doppler measurements, reduced with Earth orientation information referred to the IERS celestial system, lead to a position determination in the extragalactic reference frame with  $\sim 20$  mas accuracy. VLBI observations of the spacecraft with respect to an extragalactic radio source directly measures one component of the spacecraft position in the extragalactic frame with  $\sim 5$  mas precision (Border et al, 1982). Using these position measurements to their limiting accuracy requires knowledge of the position of the target planet in the same reference frame.

The theoretical foundation for the frame tie determination is described in sections II and III. Section II examines in detail the time dependent transformation between terrestrial and celestial frames while section III provides an analytical expression for calculating the extragalactic-planetary frame tie from the relative orientation of the VLBI and LLR terrestrial frames, and from parameters of the two VLBI and LLR terrestrial-celestial transformations. Section IV presents the analysis method and modeling used in the reduction of the VLBI and LLR data. The orientations of the planetary ephemeris DE200 relative to the extragalactic frame are calculated in section V and compared to previous determinations based on VLBI observations of spacecraft at other planets and comparison of VLBI and timing positions of millisecond pulsars. Finally, section VI presents a calculation of the location of the mean equinox at epoch J2000 in the extragalactic frame adopted by IERS.

## II. Relating Celestial and Terrestrial Frames

In the process of reducing LLR or VLBI data, a time dependent transformation between implicitly defined celestial and terrestrial coordinate frames is established. This transformation represents a dynamical tie between Earth-fixed and space-fixed frames and includes estimated and assumed precession, nutation, sidereal time, and polar motion parameters. To employ this transformation, it must be understood in some detail. To this end, the standard representation of the orientation of the Earth with respect to a celestial system is presented here. Particular attention is paid to the quantities that are commonly estimated and how they affect this representation.

Let  $\vec{x}$  represent the vector from the center of the Earth to a station in an Earth-fixed equatorial coordinate system with x-axis nominally aligned with the Greenwich meridian.\* In the fixed celestial coordinate system, the station vector  $\vec{c}$  at time  $t$  is calculated as

$$\vec{c} = \text{PN} \text{SO} \vec{x} . \quad (1)$$

The (polar motion) rotation O corrects for the offset between the Earth-fixed coordinate pole and the 'Celestial Ephemeris Pole'. The 'Celestial Ephemeris Pole' (CEP) is conceptually defined as the axis which, in the theory of the rotation of the Earth, has no forced daily or semi-daily nutations (Seidelmann, 1982; Capitaine et al., 1985). S models the rotation of the Earth about the CEP, N accounts for the quasi-periodic nutation of the CEP about the 'mean pole of date', and P models the precession or secular drift of the 'mean pole of date' and 'mean equinox of date' with respect to the fixed celestial pole and equinox of J2000. Each of these rotations is discussed in detail below.

At a particular time only three rotation angles are required to specify the rotation matrix between terrestrial and celestial coordinates. The evolution of the three angles with time is complex and conventionally the total rotation matrix is broken up into the sequence of rotations in equation (1). Nine angles are involved: three for precession and obliquity, two for nutation, two for polar motion, and UT1 -UTC. When comparing the rotation between terrestrial and celestial coordinates for VLBI and LLR, the total rotation matrices will be compared.

---

\* The time dependence of the Earth-fixed location due to tectonic motion is ignored throughout this section.

### A. Notation

In order to discuss modeling of Earth's rotation in detail, a notation for positive rotations  $\mathbf{R}_x, \mathbf{R}_y$ , and  $\mathbf{R}_z$  about the  $x, y$ , and  $z$  axes respectively is introduced, where  $\mathbf{R}_x, \mathbf{R}_y$ , and  $\mathbf{R}_z$  are defined by

$$\begin{aligned}\mathbf{R}_x(\theta) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix} \\ \mathbf{R}_y(\theta) &= \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix} \\ \mathbf{R}_z(\theta) &= \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\end{aligned}\quad (2)$$

These rotations are positive in the sense that they represent a transformation between two coordinate systems with the final coordinate system's basis vectors being rotated from the initial system's basis vectors by a right-handed rotation of angle  $\theta$  about the designated axis.

A rotation about an arbitrary axis will be defined by

$$\mathbf{R}(\vec{\Theta})\vec{r} = \vec{r} - \sin \theta \hat{\Theta} \times \vec{r} + (1 - \cos \theta) \hat{\Theta} \times (\hat{\Theta} \times \vec{r}) \quad (3)$$

where  $\theta = |\vec{\Theta}|$  is the angle of rotation,  $\hat{\Theta} = \vec{\Theta}/\theta$  is the rotation axis, and  $\vec{r}$  is an arbitrary coordinate vector. For example, with this notation  $\mathbf{R}_x(\theta) = \mathbf{R}(\theta \hat{e}_x)$ ,  $\mathbf{R}_y(\theta) = \mathbf{R}(\theta \hat{e}_y)$ , and  $\mathbf{R}_z(\theta) = \mathbf{R}(\theta \hat{e}_z)$ , where  $\hat{e}_x, \hat{e}_y$ , and  $\hat{e}_z$  are the  $x, y$ , and  $z$  unit vectors respectively. Two results will prove useful in connection with this notation. First, if  $\mathbf{M}$  is a rotation matrix, then it can be shown that

$$\mathbf{M}\mathbf{R}(\vec{\Theta})\mathbf{M}^{-1} = \mathbf{R}(\mathbf{M}\vec{\Theta}) \quad (4)$$

This follows from equation (3) and the invariance of the cross product under orthonormal coordinate transformations ( $\mathbf{M}[\vec{A} \times \vec{B}] = [\mathbf{M}\vec{A}] \times [\mathbf{M}\vec{B}]$ ). A second result is the approximation rule for small rotation vectors,

$$\mathbf{R}(\vec{B})\mathbf{R}(\vec{A}) \approx \mathbf{R}(\vec{A} + \vec{B} + \vec{A} \times \vec{B}/2) \quad (5)$$

which is accurate through second order.

### D. Terrestrial Pole orientation

The first rotation applied in the transformation from terrestrial to celestial coordinates is the orientation matrix  $\mathbf{O}$ , which accounts for polar motion, the offset of the 'Celestial Ephemeris Pole' from the terrestrial coordinate system pole:

$$\mathbf{O}(t) = \mathbf{R} \left( \begin{pmatrix} y \\ x \\ \theta \end{pmatrix} \right) \quad (6)$$

The angles  $x$  and  $-y$  are approximately the  $x$  and  $y$  coordinates of the 'Celestial Ephemeris Pole' in the Earth-fixed system.

### C. Rotation About the Pole

The vast majority of the rotational velocity of the Earth is modeled in the spin matrix

$$\mathbf{S}_0(t) = \mathbf{R}_Z(-\theta_G) \quad (7)$$

where  $\theta_G$  is the ‘Greenwich Mean Sidereal Time’, the hour angle between the meridian containing both the terrestrial x-axis and the ‘Celestial Ephemeris Pole’, and the meridian containing both this pole and the ‘mean equinox of date’. The ‘equation of the equinoxes’, which is normally included with the spin rotation  $\mathbf{S}$ , will be incorporated below in the nutation matrix  $\mathbf{N}$ . It should be noted that small rotational velocities occur due to precession, nutation, and polar motion, and therefore the ‘Celestial Ephemeris Pole’ is not the rotation axis of the Earth’s crust. By definition, the Earth-rotation-based time scale  $UT1$  is directly related to  $\theta_G$ , with the explicit relationship given by Aoki et al. (1982). When a correction to  $UT1$  is estimated, the spin matrix may be represented as

$$\mathbf{S}(t) = \mathbf{S}_0(t)\mathbf{R}_Z(-\Omega[\delta UT1]) \quad (8)$$

where  $\Omega$  is the mean rotation rate of the Earth,  $\delta UT1$  is the correction to  $UT1$ , and  $\mathbf{S}_0(t)$  is the value of  $\mathbf{S}(t)$  obtained using the nominal value of  $UT1$ .

### D. Nutation

Nutation describes the ‘short’ term quasi-periodic variations in the ‘Celestial Ephemeris Pole’. The largest term has an 18.6 year period with an amplitude of 9″ (9 arcseconds). The standard model for nutation is given by

$$\mathbf{N}(t) = \mathbf{R}_X(-\epsilon)\mathbf{R}_Z(\Delta\psi)\mathbf{R}_X(\epsilon + \Delta\epsilon)\mathbf{R}_Z(-\alpha_E) \quad (9)$$

where  $\Delta\psi$  is the nutation in longitude and  $\Delta\epsilon$  the nutation in obliquity. Here the ‘equation of the equinoxes’ rotation, which transforms from the ‘true equinox of date’ to the ‘recall equinox of date’ is given by

$$\alpha_E = \Delta\psi \cos(\epsilon) \quad (10)$$

and has been included with the nutation since the equation of the equinoxes depends only on nutation parameters. Fourier series for  $\Delta\psi$  and  $\Delta\epsilon$  for the standard (IAU 1980) model are given by Wahr (1981) or Seidelmann (1982).

To provide a better understanding of the nutation matrix, an approximate formula for it will be derived using equations (4) and (5). First, using equation (5), the nutations can be grouped together:

$$\mathbf{N}(t) \approx \mathbf{R}_X(-\epsilon)\mathbf{R} \left( \begin{pmatrix} \Delta\epsilon \\ 0 \\ \Delta\psi \end{pmatrix} \right) \mathbf{R}_X(\epsilon)\mathbf{R}_Z(-\alpha_E) \quad (11)$$

The nutations  $\Delta\psi$  and  $\Delta\epsilon$  are applied in ecliptic coordinates, with  $\mathbf{R}_X(c)$  representing a transformation into ecliptic coordinates, and  $\mathbf{R}_X(-\epsilon)$  representing a transformation back to equatorial coordinates. By using equation (4) with  $\mathbf{R}_X(-\epsilon)$  as  $\mathbf{M}$ , the nutation matrix may be reduced to a series of small rotations:

$$\mathbf{N}(t) \approx \mathbf{R} \left( \begin{pmatrix} \Delta\epsilon \\ -\Delta\psi \sin \epsilon \\ \Delta\psi \cos \epsilon \end{pmatrix} \right) \mathbf{R}_Z(-\alpha_E) \quad (12)$$

Using the explicit form (10) for the equation of the equinoxes, and collecting together small rotations using (5), the approximation

$$\mathbf{N}(t) \approx \mathbf{R} \left( \begin{pmatrix} \Delta\epsilon \\ -\Delta\psi \sin \epsilon \\ 0 \end{pmatrix} \right) \quad (13)$$

is obtained.

When nutation is estimated, the corrected nutation matrix can be represented as

$$\mathbf{N}(t) = \mathbf{N}_{IAU}(t) \mathbf{R} \left( \begin{pmatrix} \delta\epsilon \\ -\delta\psi \sin \epsilon \\ 0 \end{pmatrix} \right) \quad (14)$$

where  $\mathbf{N}_{IAU}$  is the (unapproximated) standard model and  $\delta\epsilon$  and  $\delta\psi$  are corrections to  $\Delta\epsilon$  and  $\Delta\psi$ .

#### E. Precession

Precession describes the long term drift of the ‘mean pole of date’ and ‘mean equinox of date’. The ‘mean pole’ drifts in declination by  $n \approx 20''.04$  per Julian year, and the ‘mean equinox’ drifts in right ascension by  $m \approx 46''.12$  per Julian year (Lieske, 1979). The standard model for the precession is given in the form

$$\mathbf{P}(t) = \mathbf{R}_Z(\zeta_A) \mathbf{R}_Y(-\theta_A) \mathbf{R}_Z(z_A) \quad (15)$$

Polynomial expressions for the angles  $\zeta_A$ ,  $\theta_A$ , and  $z_A$  as a function of  $t$  are given by Lieske (1979; 1977). A vectorial formulation of precession is given by Fabri (1980).

When corrections to the standard precession model are estimated, the corrected precession matrix may be represented as (Zhu and Mueller, 1983)

$$\mathbf{P}(t) = \mathbf{P}_{IAU}(t) \mathbf{R} \left( \begin{pmatrix} 0 \\ -\delta n (t - t_D) \\ \delta m \end{pmatrix} \right) \quad (16)$$

where  $\mathbf{P}_{IAU}$  is the standard model (equation (15) with Lieske’s polynomials),  $\delta n$  and  $\delta m$  are corrections to the precession rates in declination and right ascension respectively, and  $t_D$  is a reference epoch, which is preferably near the mean data epoch. The correction to the general precession in declination and right ascension,  $\delta n$  and  $\delta m$ , may be expressed as

$$\begin{aligned} \delta n &= \delta p_{LS} \sin \epsilon \\ \delta m &= \delta p_{LS} \cos \epsilon - \delta \dot{\chi} \end{aligned} \quad (17)$$

with  $\epsilon$  being the mean obliquity of the ecliptic,  $p_{LS}$  the luni-solar precession in longitude and  $\dot{\chi}$  the planetary precession in right ascension.

## F. The Total Effect of Estimated Quantities

Combining the above results and using equation (4) and equation (5) we find that the transformation from terrestrial to celestial coordinates including estimated quantities may be expressed as:

$$\mathbf{PNSO} = \mathbf{P}_{IAU} \mathbf{N}_{IAU} \mathbf{S}_0 \mathbf{R}(\vec{\Theta}) \quad (18)$$

where

$$\vec{\Theta} = \mathbf{S}_0^{-1} \mathbf{N}_{IAU}^{-1} \begin{pmatrix} 0 \\ -\delta n \\ \delta m \end{pmatrix} (t - t_D) + \mathbf{S}_0^{-1} \begin{pmatrix} \delta \epsilon \\ -\delta \psi \sin \epsilon \\ 0 \end{pmatrix} + \begin{pmatrix} y \\ x \\ -\Omega[\delta UT1] \end{pmatrix} \quad (19)$$

or, neglecting the effect of the nutation matrix on the precession corrections (this is less than 0.02 mas),

$$\vec{\Theta} = \begin{pmatrix} \delta \epsilon \cos \theta_G - (\delta n(t - t_D) + \delta \psi \sin \epsilon) \sin \theta_G + y \\ -\delta \epsilon \sin \theta_G - (\delta n(t - t_D) + \delta \psi \sin \epsilon) \cos \theta_G + x \\ \delta m(t - t_D) - \Omega[\delta UT1] \end{pmatrix} \quad (20)$$

Several things should be noted from this relationship. First, estimation of precession in declination  $\delta n$  is equivalent to estimating a linear trend in the nutation in longitude  $\delta \psi$ . In the following analysis references to  $\delta n$  are dropped in favor of a trend in  $\delta \psi$ . Second, estimation of precession in right ascension  $\delta m$  is equivalent to estimating a trend in  $\delta UT1$ . As discussed by Williams and Melbourne (1982), when corrections to precession are adopted in the future, the definition of UT1 should be altered so that UT1 series are continuous. Guinot (1979) has gone further and suggested that, rather than referring UT1 to the meridian of the mean equinox, it should be referred to a 'Non Rotating origin' which is defined on the instantaneous Earth equator in such a way as to be largely independent of precession and nutation models (Capitaine et al. 1986). In the light of this thinking the precession in right ascension  $\delta m$  is not a useful parameter and cannot be estimated. Finally, it should be noted that, on time scales short compared to a day, it is impossible to distinguish between nutation and polar motion; only three angles are needed to describe a general rotation. In fact, if nutations were allowed to have rapid variations with nearly daily periods, there would be no need for the polar angles  $x$  and  $y$ . This points to the fact that, whatever the conceptual definition of the 'Celestial Ephemeris Pole', its present implementation results from fitting data to slowly varying nutation and polar motion models.

## III. Relating the VLBI, LLR, and Planetary Ephemeris Reference Frames

In this section, the relationships between the planetary ephemeris and the LLR and VLBI reference frames are discussed. The end result is an expression relating the extragalactic-planetary frame tie to quantities available from the LLR and VLBI data reductions and to the transformation between LLR and VLBI terrestrial frames. The extragalactic-planetary frame tie is represented by a rotation vector  $\vec{A}$  that relates VLBI celestial coordinates  $\vec{c}_{VLBI}$  and planetary ephemeris coordinates  $\vec{c}_{PE}$  by

$$\vec{c}_{VLBI} = \mathbf{R}(\vec{A}) \vec{c}_{PE} \quad (21)$$

The derivation of the frame tie starts with the planetary ephemeris as represented by the ephemeris of the Earth and proceeds in steps to the LLR terrestrial frame, the VLBI terrestrial frame, and finally to the VLBI celestial frame, which is tied to the celestial system adopted by IERS for astrometric and geodetic applications.

#### Transformation between the planetary ephemeris frame and the LLR terrestrial frame

In the LLR data reduction, terrestrial coordinates  $\vec{x}_{LLR}$  and celestial coordinates in the planetary ephemeris frame  $\vec{c}_{PE}$  are related by

$$\vec{c}_{PE} = \mathbf{R}(\delta P_B \hat{P}_B + \delta Q_B \hat{Q}_B) \mathbf{P}_{IAU} \mathbf{N}_{IAU} \mathbf{S}_0 \mathbf{R}(\vec{\Theta}_{LLR}) \vec{x}_{LLR} \quad (22)$$

where  $\vec{\Theta}_{LLR}$  has the form of equation (20), and where  $\delta P_B$  and  $\delta Q_B$  are corrections to the orientation of the Earth's orbit estimated in the LLR analysis.  $\hat{P}_B$  is a unit vector in the direction of the Earth's perihelion and  $\hat{Q}_B$  is a unit vector in a direction orthogonal to both the direction to perihelion and to the ecliptic pole (see Brouwer and Clemence 1961).  $\hat{P}_B$  and  $\hat{Q}_B$  are given by:

$$\hat{P}_B = \begin{pmatrix} \cos \omega \\ \sin \omega \cos \epsilon \\ \sin \omega \sin \epsilon \end{pmatrix} \quad \hat{Q}_B = \begin{pmatrix} -\sin \omega \\ \cos \omega \cos \epsilon \\ \cos \omega \sin \epsilon \end{pmatrix} \quad (23)$$

with the argument of perihelion  $\omega = 102.94^\circ$  at J 2000.

#### Transformation between the LLR and VLBI terrestrial frames

The complete transformation from LLR to VLBI terrestrial coordinates must account for a rotation, a translation, and a possible difference of scale. Here our main concern is with the relative orientation of the two frames. Therefore, the relationship between coordinates in the two frames will be represented as

$$\vec{x}_{VLBI} = \mathbf{R}(\vec{R}) \vec{x}_{LLR} \quad (24)$$

where the vector  $\vec{R}$  parameterizes the rotation between the LLR and VLBI terrestrial frames. The method used to derive this rotation will be explained in Section V.

#### Transformation between the VLBI terrestrial and celestial frames

In the VLBI data analysis, terrestrial coordinates  $\vec{x}_{VLBI}$  and celestial coordinates  $\vec{c}_{VLBI}$  are related by

$$\vec{c}_{VLBI} = \mathbf{R}(\vec{\Theta}_P) \mathbf{P}_{IAU} \mathbf{N}_{IAU} \mathbf{S}_0 \mathbf{R}(\vec{\Theta}_{VLBI}) \vec{x}_{VLBI} \quad (25)$$

where  $\vec{\Theta}_{VLBI}$  has the form of equation (20) and where  $\Theta_P$  is a small incremental rotation carrying the mean CEP pole of year 2000 into the pole of our VLBI frame, coincident with that of the IERS frame (see Section IV). The rotation vector  $\vec{\Theta}_P$  is represented by:

$$\vec{\Theta}_P = \begin{pmatrix} \theta_1 \\ \theta_2 \\ 0 \end{pmatrix} \quad (26)$$

where  $(-\hat{z}, \theta_1, 1)$  are the coordinates of the mean CEP pole of year 2000 in the VLBI/IERS frame.

### Expression of the extragalactic-planetary frame tic

Tracing the coordinate transformation

$$\vec{c}_{VLBI} \leftarrow \vec{x}_{VLBI} \leftarrow \vec{x}_{LLR} \leftarrow \vec{c}_{PE}$$

the extragalactic-planetary frame tic is found to be given by

$$\begin{aligned} \mathbf{R}(\vec{A}) = & \mathbf{R}(\vec{\Theta}_P) \mathbf{P}_{IAU} \mathbf{N}_{IAU} \mathbf{S}_0 \mathbf{R}(\vec{\Theta}_{VLBI}) \mathbf{R}(\vec{R}) \mathbf{R}(-\vec{\Theta}_{LLR}) \mathbf{S}_0^{-1} \mathbf{N}_{IAU}^{-1} \mathbf{P}_{IAU}^{-1} \\ & \times \mathbf{R}(-\delta P_B \hat{P}_B - \delta Q_B \hat{Q}_B) \end{aligned} \quad (27)$$

Using equation (4) and equation (5) this reduces to

$$\mathbf{R}(\vec{A}) = \mathbf{R}(\vec{\Theta}_P) \mathbf{R}(\mathbf{P}_{IAU} \mathbf{N}_{IAU} \mathbf{S}_0 [\vec{\Theta}_{VLBI} + \vec{R} - \vec{\Theta}_{LLR}]) \mathbf{R}(-\delta P_B \hat{P}_B - \delta Q_B \hat{Q}_B) \quad (28)$$

and then

$$\vec{A} = \vec{\Theta}_P + \mathbf{P}_{IAU} \mathbf{N}_{IAU} \mathbf{S}_0 [\vec{\Theta}_{VLBI} + \vec{R} - \vec{\Theta}_{LLR}] - \delta P_B \hat{P}_B - \delta Q_B \hat{Q}_B \quad (29)$$

Neglecting the effect of precession and nutation on small quantities (this is less than 0.1 mas and is not significant in our analysis), the components of the rotation vector  $\vec{A}$  are given by

$$\begin{aligned} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = & \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} + \begin{pmatrix} \delta\epsilon_{VLBI} - \delta\epsilon_{LLR} \\ -(\delta\psi_{VLBI} - \delta\psi_{LLR}) \sin \epsilon \end{pmatrix} \\ & + \begin{pmatrix} \cos \theta_G & -\sin \theta_G \\ \sin \theta_G & \cos \theta_G \end{pmatrix} \begin{pmatrix} R_1 + y_{VLBI} - y_{LLR} \\ R_2 + x_{VLBI} - x_{LLR} \end{pmatrix} \\ & + \begin{pmatrix} -\delta P_B \cos \omega + \delta Q_B \sin \omega \\ -\delta P_B \sin \omega \cos \epsilon - \delta Q_B \cos \omega \cos \epsilon \end{pmatrix} \\ A_3 = & R_3 - \Omega(\delta UT1_{VLBI} - \delta UT1_{LLR}) - \delta P_B \sin \omega \sin \epsilon - \delta Q_B \cos \omega \sin \epsilon \end{aligned} \quad (30)$$

where the correction to precession in declination is included as a trend in  $\delta\psi$  and a correction to precession in right ascension is not distinct from a trend in  $\delta UT1$ . As argued earlier, nutation and precession can be separated from polar motion only by requiring that each be slowly varying. Therefore the terms in equation (30) that are modulated by sinusoids in  $\theta_G$ , the Greenwich Mean Sidereal Time, must be separately zero. This gives the bias between the two polar motion series in terms of the terrestrial transformation parameters  $R_1$  and  $R_2$ , which correspond to a displacement of the coordinate pole:

$$\begin{aligned} y_{LLR} - y_{VLBI} &= R_1 \\ x_{LLR} - x_{VLBI} &= R_2 \end{aligned} \quad (31)$$

The full three dimensional frame tic rotation vector is then given by

$$\vec{A} = \begin{pmatrix} \theta_1 + \delta\epsilon_{VLBI} - \delta\epsilon_{LLR} - \delta P_B \cos \omega + \delta Q_B \sin \omega \\ \theta_2 - (\delta\psi_{VLBI} - \delta\psi_{LLR}) \sin \epsilon - \delta P_B \sin \omega \cos \epsilon - \delta Q_B \cos \omega \cos \epsilon \\ R_3 - \Omega(\delta UT1_{VLBI} - \delta UT1_{LLR}) - \delta P_B \sin \omega \sin \epsilon - \delta Q_B \cos \omega \sin \epsilon \end{pmatrix} \quad (32)$$

The calculation of  $\vec{A}$  generally requires a comparison of LLR and VLBI nutation and universal time estimates, a determination of the transformation between VLBI and LLR terrestrial frames, and knowledge of corrections to the orientation of the Earth's orbit and to the position of the mean CEP pole of epoch J2000 estimated in the analysis.

#### IV. The joint VLBI/LLR analysis

In the past decade, software for the reduction of VLBI observations (Sewers 1991b) and LLR observations has been developed at the Jet Propulsion Laboratory (JPL). The software formulation allows the two data types to be combined for estimation of parameters in common to VLBI and LLR. Such a combined analysis was initially developed to produce joint VLBI/LLR estimates of precession and nutation constants (Chariot et al. 1991). The combined data set is stronger than either individually because the LLR data set is of longer duration (beginning in 1969) while the VLBI data are currently more accurate and more frequent. This combined analysis is also useful for determining the extragalactic planetary frame because consistency of VLBI and LLR Earth orientation modeling inherent in the method provides simplification of equation (32) for the frame tie calculation.

The VLBI data used in our analysis consist of about 10 000 delay and delay rate pairs acquired during 127 dual-frequency (2.3 and 8.4 GHz) VLBI observing sessions carried out by the NASA's Deep Space Network (DSN) between October 1978 and February 1993 on two intercontinental baselines: Goldstone-Madrid and Goldstone-Tidbinbilla. Approximately half of these observations (starting in 1988) were recorded with the Mark 111 data acquisition system (Rogers et al. 1983) with a precision of 1 cm. The older observations, recorded with the Mark II system (Clark 1973), have a precision of 10 cm.

The LLR data consist of about 8000 ranges (distance measurements) from telescopes on the Earth to reflectors on the Moon acquired between August 1969 and February 1993. The measurements of the first decade are all from the 2.7 m telescope at McDonald Observatory (Texas), which ceased LLR activity in 1985. Those of the past decade are from three telescopes, located in Grasse (France), Haleakala (island of Maui, Hawaii), and at McDonald Observatory, which are dedicated to ranging the Moon and Earth-orbiting satellites. The current precision of LLR measurements is about 3 cm while the precision of early data was 30 cm.

In the combined VLBI/LLR analysis, corrections to periodic nutation terms were estimated in common to both data types as well as a linear correction to nutation in longitude (equivalent to a correction to luni-solar precession). Such parameters are necessary because the IAU (1976) precession constant is known to be in error by several mas/yr and the IAU (1980) theory of nutation amplitudes are in error by several mas (e.g. Herring et al. 1991, Williams et al. 1991). In addition, UT1 tidal amplitudes at the nearly diurnal ( $K_1, P_1, O_1$ ) and semi-diurnal ( $S_2, M_2, N_2$ ) frequencies were also jointly estimated because the present theoretical models available are not accurate enough to fit the VLBI data (e.g. Sovers et al. 1993). Offsets to the a priori polar motion/UT1 series (Gross, 1993) were constrained to be zero. Corrections to UT1 and polar motion rates were estimated separately for each data type since station velocities were set to agree with the NNR-NUVEL-1 plate tectonic model (De Mets et al. 1990, Argus and Gordon 1991). Table 1 gives the estimated values for the nutation parameters, UT1 tidal amplitudes, and VLBI and LLR polar motion and UT1 rates. The parameter uncertainties given in Table 1 and throughout, the rest of this section represent the formal errors of the least squares fit. The formal uncertainties are known to be smaller than the actual uncertainties due to several effects not included in the least-squares fit. A better representation of the actual parameter uncertainties is given in the next section for parameters important in the derivation of the frame tie.

In order to fit the VLBI data with the above Earth orientation modeling, the locations of the DSN radio telescopes were estimated along with relative coordinates of extragalactic radio sources and session-specific clock and troposphere parameters for each station. The relative locations of the various radio telescopes within each site (Goldstone, Madrid, and Tidbinbilla) were constrained (to 1 cm) by intra-site VLBI experiments (Jacobs, 1993). Table 2 gives the estimated VLBI radio telescope positions. The VLBI celestial frame was tied to the IERS celestial system by fixing the right ascension and declination of OJ287 (0851+202) and the declination of CTD20 (0234+285) to their values in the ICRF91 frame, as devised by Steppe et al. (1992). The position of the mean CEP pole of year 2000, defined in equation (26), was also estimated, and its coordinates were  $\theta_1 = -1.5 \pm 0.2$  mas and  $\theta_2 = 21.63 \pm 0.5$  mas. The large offset of the CEP position about the y-axis,  $19_2$ , is mainly due to the linear correction to nutation in longitude obtained in our combined analysis ( $-3.79$  mas/yr). This correction is consistent with that found by other groups (e.g.  $-3.2 \pm 1.3$  mas/yr, Herring et al. 1991).

LLR-specific parameters estimated included station locations, reflector locations, corrections to the orbit of the Earth-moon barycenter about the sun, as tabulated in the experimental planetary and lunar ephemeris DE21 0/LE2 10, and lunar gravity, rotation and orbit parameters. The corrections to the Earth's orbit were found to be  $\delta P_B = 4.0 \pm 0.6$  mas and  $\delta Q_B = -2.4 \pm 0.3$  mas. The estimated LLR site locations are listed in Table 3.

Equation (32) shows that the extragalactic-planetary frame tie generally depends on the offsets,  $\delta UT1_{VLBI} - \delta UT1_{LLR}$ ,  $\delta \epsilon_{VLBI} - \delta \epsilon_{LLR}$ , and  $\delta \psi_{VLBI} - \delta \psi_{LLR}$ , between the VLBI and LLR Earth orientation corrections. In our combined VLBI/LLR analysis, these offsets cancel because the Earth orientation was made consistent for VLBI and LLR. Thus, the frame tie calculation is simplified. The VLBI and LLR rate corrections to the a priori UT1 and polar motion X are insignificant (see Table 1). The rate corrections to polar motion Y are significant and disagree for LLR and VLBI. The VLBI polar motion Y rate offset ( $0.26 \pm 0.04$  mas/yr) is consistent with the known rate offset ( $0.15 \pm 0.04$  mas/yr) between the IERS polar motion series to which our a priori series is tied (see Gross 1993) and the IERS terrestrial reference frame (IERS, 1993). The larger difference in the LLR polar motion Y rate may be due to a difference between the LLR estimated values and the a priori series for times prior to 1984, where no VLBI data exist, or possibly due to a discrepancy between the NNR-NUVEL-1 plate motion and the actual site velocity at one of the three LLR sites. At this level of discrepancy, however, a polar motion Y rate should not significantly affect the frame tie determination.

## V. The extragalactic-planetary frame tie

After constraining Earth orientation offsets to agree in the combined VLBI/LLR analysis, the frame tie may be computed in terms of the coordinates of the CEP in the extragalactic frame, the corrections to the planetary ephemeris, and the relative orientation of the VLBI and LLR, terrestrial frames about the z-axis. Estimates for all of these parameters except the terrestrial frame orientation are given above. A direct comparison of the VLBI and LLR terrestrial frames is not possible since DSN VLBI sites and LLR stations are widely separated. Fortunately, recent efforts to unify VLBI, LLR and SLR (satellite laser ranging) terrestrial frames, both by the International Earth Rotation Service (Boucher and Altamimi 1989) and by independent groups (Ray et al. 1991), have resulted in a terrestrial

frame cent aining all of the DSN and LLR sites. The transformation between the VLBI and LLR terrestrial frames in the combined VLBI/LLR analysis may then be determined by comparing them with such a unified terrestrial frame. For our analysis, WC chose the 1992 realization of the IERS terrestrial frame labeled 1 TRF92 (Boucher et al. 1993). Table 4 gives the VLBI and LLR station coordinates the ITRF92 system as given by (Boucher et al. 1993).

The individual VLBI and LLR frames have been compared to the ITRF92 frame with a seven-parameter transformation (3 translations, 3 rotations, one scale factor) given by:

$$\begin{aligned}\vec{x}_{VLBI} &= \vec{T}' + (1 + D')\vec{x}_{ITRF92} + \mathbf{R}(\vec{R}')\vec{x}_{ITRF92} \\ \vec{x}_{LLR} &= \vec{T}'' + (1 + D'')\vec{x}_{ITRF92} + \mathbf{R}(\vec{R}'')\vec{x}_{ITRF92}\end{aligned}\quad (33)$$

where  $\vec{x}_{VLBI}$ ,  $\vec{x}_{LLR}$ ,  $\vec{x}_{ITRF92}$  are coordinate vectors in the VLBI, LLR and ITRF92 frames,  $\vec{T}'$  and  $\vec{T}''$  ( $\vec{R}'$  and  $\vec{R}''$ , respectively) are translations (rotations, respectively) from the ITRF92 frame to the VLBI and LLR frames, and  $D'$  and  $D''$  are scale factors from the ITRF92 frame to the VLBI and LLR frames. The seven parameters of each transformation have been estimated with a least-squares analysis comparing the station coordinates of each frame (VLBI or LLR) to the corresponding station coordinates in the ITRF92 frame. Table 5 shows the values of these parameters. The relative orientation of the VLBI and LLR terrestrial frames about the  $z$ -axis needed for the frame tic calculation may be derived from the parameters given in Table 5 as  $R_3 = -39.4 \pm 1.9$  mas.

Returning to equation (32) and substituting the terrestrial rotation  $R_3$ , corrections to the orientation of the Earth's orbit,  $\delta P_B$  and  $\delta Q_B$ , and corrections to the position of the mean CEP pole of year 2000,  $\theta_1$  and  $\theta_2$ , given above, provides the following estimate of the tic from the frame of the experimental planetary ephemeris DE210 to the IERS celestial reference frame.

$$A_{DE210}^{IERS} = \begin{pmatrix} -3 + 2 \text{ mas} \\ 18 \pm 4 \text{ mas} \\ -41 \pm 3 \text{ mas} \end{pmatrix} \quad (34)$$

The uncertainties printed in equation (34) are not formal errors. For their determination, we have multiplied the formal errors of  $\delta P_B$  and  $\delta Q_B$  by a factor of 5, accounting for the fact that the mutual inclinations of the ecliptic, the equator, and the lunar orbital plane are not known to better than 2 mas accuracy (Williams and Standish 1989). Similarly, the formal errors of  $\theta_1$  and  $\theta_2$  have also been enlarged by a factor of 5, assuming an uncertainty of 0.5 mas/yr in the precession constant (the formal error was only 0.1 mas/yr). The uncertainty in  $R_3$  (+3 rms) accounts for likely systematic errors due to the limited number of sites in the station coordinate comparison.

The frame tic given in equation (34) is dependent on the data span used in the least-squares fit and is most valid at the time of the weighted mean data epoch, which is about 1988. This is because the frame tic is sensitive to the positions of the planets relative to the radio sources at the time LLR and VLBI data exist while the uncertainties of the positions of

the planets at times relative to the 1988 generally increase with time. In particular, the mean motion of the Earth-moon barycenter about the sun, determined primarily by Viking lander ranging, is uncertain by  $\sim 0.15 \text{ mas/yr}$  due to uncertainties in the masses of the asteroids Ceres, Pallas, and Vesta (Williams, 1993). An analysis of a similar LLR/VLBI data set with a mean data epoch ten years later could be expected to give a result different by  $\sim 1.5 \text{ mas}$ .

The frame tie between the older planetary ephemeris DE200 (Standish 1982,1990) which was extensively used in the past decade, and the IERS celestial frame,  $\vec{A}_{DE200}^{IERS}$ , can also be inferred from our analysis since the relative rotation vector between DE200 and DE210 can be calculated by comparing the orbit of the Earth-moon barycenter in the two ephemerides. Such a comparison gives a time-dependent rotation since the two ephemerides are based on different data sets. The ephemeris DE200 included only part of the Viking lander data and the masses of the asteroids were less certain at the time of the DE200 solution. The rotation between DE200 and DE210 is found to drift by  $\sim 0.5 \text{ mas/yr}$  which is greater than the current uncertainty of the Earth's mean motion about the sun. For the mean data epoch of 1988, the rotation between DE200 and DE210 is approximately given by  $\vec{A}_{DE200}^{DE210} = (1 \text{ mas}, -30 \text{ mas}, 35 \text{ mas})$ . Our estimate of the frame tie between the planetary ephemeris DE200 and the IERS celestial frame is then:

$$\vec{A}_{DE200}^{IERS} = \begin{pmatrix} -2 \pm 2 \text{ mas} \\ -12 \pm 4 \text{ mas} \\ -6 \pm 3 \text{ mas} \end{pmatrix} \quad (35)$$

This frame tie is consistent with that determined by Finger and Folkner (1992) with a similar method ( $\vec{A} = 1 \pm 3 \text{ mas}, -10 \pm 3 \text{ mas}, -4 \pm 5 \text{ mas}$ ). It can also be compared to results obtained by other techniques by examining the offset in right ascension and declination in the part of the sky where the other measurements exist, keeping in mind that a largely-known drift exists for data reduced with respect to DE200 of  $\sim 0.5 \text{ mas/yr}$ .

There have been a number of VLBI observations of spacecraft at other planets. A planetary orbiter, or a spacecraft making a planetary encounter, has a position determined with respect to the planet from the gravitational signature on the spacecraft Doppler data. VLBI measurements between the spacecraft and one or more angularly nearby radio sources can be used to estimate the radio source coordinates in the planetary reference frame. Newhall et al. (1986) reported average right ascension and declination offsets consistent with zero with uncertainty 8-12 mas based on the results of VLBI measurements for the Viking and Pioneer Venus orbiters. This determination is consistent with our result in equation (35). McElrath and Bhat (1988) derived a position of the radio source 0202 - 149 in the frame of the planetary ephemeris DE200 from observations of the Soviet Vega 1 and Vega 2 spacecraft as they flew by Venus in 1985. The uncertainty in this determination is dominated by the uncertainty in the location of Venus with respect to the Earth's orbit. In Table 6 the results of the Vega flyby measurements are compared with our determination and are seen to be consistent within one sigma. In 1989, two single-baseline VLBI observations of the Soviet Phobos II spacecraft at Mars were made to determine one component of the positions of two radio sources in the frame of DE200 (Iijima and

Hildebrand 1991 ). The results of these two measurements are consistent with the result of equation (35) as shown in Table 6.

Timing of millisecond pulsars gives positions with accuracy better than 1 mas based on the orbit of the Earth (Rawley et al. 1988). VLBI observations of these sources are difficult since the pulsars are weak radio sources. Two groups (Dewey et al., 1991; Bartel, 1991) have made VLBI observations of the pulsar PSR 1937+21. Preliminary results from one group (Dewey et al. 1991), when combined with the results of the timing measurements, give right ascension and declination offsets for the pulsar that are in agreement with our frame tic determination (see Table 6).

In the future more spacecraft VLBI measurements and refinements of the technique presented here, as well as results from other methods, should combine to produce a consistent, frame tic determination at the 1 mas level. In the meantime, the extragalactic-planetary frame tic result presented here with 3 mas accuracy is the most accurate currently available.

## VI. Location of the mean equinox of J2000 in the IERS system

It has been traditional for astrometry to refer to a coordinate system defined by the mean equator and equinox of a reference epoch (e.g. B 1950, J2000), where the mean equinox of epoch is the intersection of the mean equator with the mean ecliptic. VLBI data allow the determination of the mean equator of J2000 in the IERS celestial system with an accuracy of a few mas. By combining this result with a determination of the mean ecliptic based on analysis of the planetary ephemeris and by using the frame tic calculated in Section V, it is then possible to determine the location of the mean equinox of J2000 in the IERS system.

The unit vector in the direction of the mean equinox at J2000 in the IERS celestial frame,  $\hat{\gamma}_{IERS}$ , can be defined as the cross product of a unit vector pointing toward the mean equatorial pole of J2000,  $\hat{P}_{equ}^{IERS}$ , with a unit vector pointing toward the mean ecliptic pole of J2000,  $\hat{P}_{ecl}^{IERS}$ :

$$\hat{\gamma}_{IERS} = \frac{1}{\sin \epsilon} (\hat{P}_{equ}^{IERS} \times \hat{P}_{ecl}^{IERS}) \quad (36)$$

The unit vector towards the mean ecliptic pole in the IERS celestial frame, is obtained from equation (26):

$$\hat{P}_{equ}^{IERS} = \begin{bmatrix} -0_2 \\ \theta_1 \\ 1 \end{bmatrix} \quad (37)$$

The mean ecliptic pole was determined in the process of creating the ephemeris DE200 in an attempt to orient that ephemeris with the mean equator and equinox of 2000 (Standish, 1982). That calculation is still useful for the present work because the determination of the mean ecliptic pole (unlike the determination of the mean equator) is independent of precession and nutation models. Since the ephemeris DE200 was aligned such that the

node of the ecliptic was zero at the epoch J2000, the mean ecliptic pole of J2000 in the reference frame of DE200, is given by the unit vector:

$$\hat{P}_{ecl}^{DE200} = \begin{pmatrix} 0 \\ -\sin \epsilon \\ \cos \epsilon \end{pmatrix} \quad (38)$$

with  $\epsilon = 23^\circ 26' 21''.412$  (Standish 1982). Note that this determination of the ecliptic pole is referred to a rotating ecliptic (Standish, 1981). The mean ecliptic pole can be rotated by using equation (21) to find its coordinates in the IERS frame:

$$\hat{P}_{ecl}^{IERS} = \begin{pmatrix} -A_2 \cos \epsilon - A_3 \sin \epsilon \\ -\sin \epsilon + A_1 \cos \epsilon \\ \cos \epsilon + A_1 \sin \epsilon \end{pmatrix} \quad (39)$$

where  $A_1, A_2, A_3$  are the components of the frame tic rotation vector,  $\vec{A}_{DE200}^{IERS}$ , given in equation (35). The expression of the mean equinox vector,  $\hat{\gamma}_{IERS}$ , is then derived from equation (36) as:

$$\hat{\gamma}_{IERS} = \begin{pmatrix} 1 \\ -A_3 + (\theta_2 - A_2)/\tan \epsilon \\ \theta_2 \end{pmatrix} \quad (40)$$

By using values of  $\theta_2, A_2$  and  $A_3$  from Sections IV and V, the coordinates in the IERS frame of the mean equinox vector at epoch J2000 have been calculated. They are given in the equation below.

$$\hat{\gamma}_{IERS} = \begin{pmatrix} 1 \\ 83 \pm 10 \text{ mas} \\ 223.3 \text{ mas} \end{pmatrix} \quad (41)$$

The accuracy of this determination is limited by the averaging procedure used to determine the mean ecliptic. The longest period term used in the numerical fit for DE200 had a period of 882 years. A comparison analytic theories of the motion of the planets with the numerically integrated DE200 give values of the inclination of the mean ecliptic to the ecliptic of date consistent with Standish's at the mas level (Bretagnon, 1982; Chapront-Touze and Chapront, 1983). However, a more recent analysis of numerically integrated ephemerides, which included a term with a period of 1781 years gave a shift in the mean ecliptic pole of 2 mas. Standish, 1993). The uncertainties given in equation (41) reflect the latter study and our caution in giving results with respect to the mean ecliptic,

## VII. Conclusion

The rotational offset between the JPL planetary ephemeris DE200 defined by the orbits of the Earth and the planets and the VI, I/ IERS extragalactic frame has been determined

with 3 mas accuracy from comparison of VLBI and LLR station coordinates derived from a joint VLBI/LLR analysis. This result, which is the most accurate frame tie determination obtained so far, will enable more accurate interplanetary spacecraft navigation. The frame tie is dependent on the time of the mean data epoch because of uncertainties internal to the planetary ephemeris. Comparison of this frame tie result with results obtained by other techniques, e.g. VLBI observations of spacecraft at other planets, or comparison of VLBI and timing positions of millisecond pulsars, indicate that our determination is consistent with those previous results. The location of the mean equinox of J2000 in the IERS intragalactic frame has also been calculated from our combined VLBI/LLR analysis. Our calculation shows that the mean equinox of J2000 is shifted from the  $x$ -axis of the IERS system by 834.10 mas along the  $y$ -direction and by  $22 \pm 3$  mas along the  $z$ -direction. The limiting factors of this determination are the knowledge of the precession constant and the relative orientation of VLBI and LLR terrestrial frames, and the averaging procedure used to calculate the mean ecliptic. In the future, improved precession and nutation constants and a larger LLR network should lead to an accuracy of 1 mas in the calculation of the extragalactic-planetary frame tie with this method.

#### Acknowledgments

We are grateful to Z. Altamimi for providing the ITRF92 frame before publication and for useful discussions. This work would not have been possible without the efforts of many people involved in the acquisition and processing of DSN VLBI data and lunar laser ranging data. The research described in this paper was carried out in part at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

## References

- S. Aoki, B. Guinot, G. H. Kaplan, H. Kinoshita, D. D. McCarthy, and P. K. Seidelmann: 1982, "The New Definition of Universal Time", *Astron. Astrophys.* **105**, 359-361.
- D. F. Argus and R. G. Gordon: 1991, "No-Net-Rotation Model of Current Plate Velocities Incorporating Plate Motion Model NUVEL-1", *Geophys. Res. Letters* **18**, 2039-2048.
- E. F. Arias, M. Feissel, and J.-F. Lestrade: 1988, "An extragalactic celestial reference frame consistent with the BIH Terrestrial System (1987)", BIH Annual Report for 1987, Observatoire de Paris, Paris, France, p. D-113.
- N. Bartel, 1991: private communication.
- J. S. Border, F. F. Donovan, S. G. Finley, C. E. Hildebrand, B. Moultrie, and L. J. Skjerve: 1982, "Determining Spacecraft Angular Position with Delta VLBI: The Voyager Demonstration", paper AIAA-82-1471, AIAA/AAS Astrodynamics Conference, San Diego, California, August 9-11, 1982.
- C. Boucher and Z. Altamimi: 1989, "The Initial IERS Terrestrial Reference Frame", IERS Technical Note 1, Observatoire de Paris.
- C. Boucher, Z. Altamimi, and L. Duhem: 1993, "ITRF92 and its associated field", IERS Technical Note 15, Observatoire de Paris.
- P. Bretagnon: 1982, "Théorie du Mouvement de l'ensemble des Planètes. Solution VSOP82", *Astron. Astrophys.* **114**, 278-288.
- J. Brouwer and G. M. Clemence: 1961, "Methods of Celestial Mechanics", Academic Press, pp. 233,
- N. Capitaine, J. G. Williams, and P. K. Seidelmann: 1985, "Clarifications concerning the definition of the celestial ephemeris pole", *Astron. Astrophys.* **146**, 381-383.
- N. Capitaine, B. Guinot, and J. Souchay: 1986, "A Non-Rotating Origin on the Instantaneous Equator: Definition, Properties, and Use", *Celestial Mechanics* **39**, 283-307.
- M. Chapront-Touze and J. Chapront: 1983, "The Lunar Ephemeris ELP 2000", *Astron. Astrophys.* **124**, 50-62.
- P. Chriot, O. J. Severs, J. G. Williams, and X. X. Newhall: 1990, "A Global VLBI/LLR Analysis for the Determination of Precession and Nutation Constants", *References Systems, Proceedings of International Astronomical Union Colloquium 127*, U. S. Naval Observatory, Washington, pp. 228-233.
- B. G. Clark: 1973, "The NRAO tape-recorder interferometer system", *Proc IEEE*, **61**, 1242-1248.
- C. DeMets, R. G. Gordon, D. F. Argus, and S. Stein: 1990, "Current Plate Motions", *Geophys. J. Int.* **101**, 425-478.
- R. J. Dewey and D. L. Jones: 1991, private communication.

- E. Falori: 1980, "Advocating the Use of Vector-Matrix Notation in Precession Theory", *Astron. Astrophys.* **82**, 123-128.
- M. H. Finger, and W.M. Folkner: 1992, "A Determination of the Radio-Planetary Frame Tie from Comparison of Earth Orientation Parameters", *TDA Progress Report 42-109*, pp. 1-21.
- R. S. Gross: 1993, "A Combination of Earth Orientation Data: SPACE92", *IERS Technical Note 14*, P. Chariot (ed.), Observatoire de Paris (in press).
- B. Guinot: 1979, "Basic Problems in the Kinematics of the Rotation of the Earth", *Time and The Earth's Rotation*, D. D. McCarthy and J. D. Pilkington eds., p. 7-18.
- C. Hazard, J. Sutton, A. N. Argue, C.M. Kenworthy, L. V. Morrison, and C. A. Murray: 1971, "Accurate Radio and Optical Positions of 3C273B", *Nature* **233**, 89-91.
- T. A. Herring, B. A. Buffett, P. M. Mathews, and I. I. Shapiro: 1991, "Forced Nutations of the Earth: Influence of Inner Core Dynamics 111. Very Long Baseline Interferometry Data Analysis", *J. Geophys. Res.* **96**, 8259-8273.
- IERS: 1993, "(1992 International Earth Rotation Service Annual Report", Observatoire de Paris.
- B. A. Iijima and C. E. Hildebrand: 1991, private communication.
- C. S. Jams: 1993, private communication.
- J. H. Lieske: 1977, "Expressions for the Precession Quantities Based upon the IAU (1976) System of Astronomical Constants", *Astron. Astrophys.* **58**, 1-16.
- J. H. Lieske, T. Lederle, W. Fricke, and B. Morando: 1979, "Precession Matrix Based on IAU (1976) System of Astronomical Constants", *Astron. Astrophys.* **73**, 283-284.
- T. P. McElrath and R. S. Bhat: 1988, "Determination of the Inner Planet Frame Tie Using VLBI Data", *Proceedings of the AIAA/AAS Astrodynamics Conference*, paper 88-4234, Minneapolis.
- X X Newhall, R. A. Preston, and P. B. Esposito: 1986, "Relating the JPL VLBI Reference Frame and the Planetary Ephemerides", *Astrometric Techniques; Proceedings of the 109th Symposium of the IAU*, 789-794, H. K. Eichorn and R. J. Leacock (eds.), Riedel, Boston.
- X X Newhall, J. G. Williams, and J. O. Dickey: 1993, "Earth Rotation (UT0-UTC and Variation of Latitude) from Lunar Laser Ranging", *IERS Technical Note 14*, P. Charlot (ed.), Observatoire de Paris (in press).
- L. A. Rawley, J. H. Taylor, and M. M. Davis: 1988, "Fundamental Astrometry and Millisecond Pulsars", *Astrophys. J.* **326**, 947-953.
- J. R. Ray, C. Ma, T. A. Clark, J. W. Ryan, R. J. Fances, M. M. Watkins, B. E. Schutz, and B. D. Tapley: 1991, "Comparison of VLBI and SLR Geocentric Site Coordinates", *Geophys. Res. Lett.* **18**, 231-234.

A. E. E. Rogers, R. J. Cappallo, H. F. Hinteregger, J. I. Levine, E. F. Nesman, J. C. Webber, A. R. Whitney, T. A. Clark, C. Ma, J. Ryan, B. E. Corey, C. C. Counselman, T. A. Herring, I. I. Shapiro, C. A. Knight, D. B. Shaffer, N. R. Vandenberg, R. Lacasse, R. Mauzy, B. Rayhrer, B. R. Schupler, and J. C. Pigg,: 1983, "Very-Long Baseline Interferometry: The Mark III System for Geodesy, Astrometry, and Aperture Synthesis", *Science* **219**, 51-54.

P. K. Seidelmann: 1982, "1980 IAU Theory of Nutation: The Final Report of the IAU Working Group on Nutation", *Celestial Mechanics* **27**, 79-106.

O. J. Sovers: 1991a, "JPL 1990-3: A 5-mrad Extragalactic Source Catalog Based on Combined Radio Interferometric observations", TDA Progress Report 42-106, pp. 364-383.

O. J. Sovers: 1991 b, "Observation Model and Parameter Partial for the JPL VLBI Parameter Estimation Software MODEST", JPL Publication 83-39, Rev. 4, Jet Propulsion Laboratory, Pasadena, California.

O. J. Sovers, C. S. Jacobs, and R. S. Gross: 1993, "Measuring Rapid Ocean Tidal Earth Orientation Variations with VLBI", *J. Geophys. Res.* (in press).

E. M. Standish Jr.: 1981, "Two Differing Definitions of the Dynamical Equinox and the Mean Obliquity", *Astron. Astrophys.* **101**, L17-L18.

E. M. Standish Jr.: 1982, "Orientation of the JPL Ephemerides, DE200/LE200, to the Dynamical Equinox of J2000", *Astron. Astrophys.* **114**, 297-302.

E. M. Standish Jr.: 1990, "The Observational Basis for JPL's DE200, the Planetary Ephemeris of the Astronomical Almanac.", *Astron. Astrophys.* **233**, 252-271.

J. A. Steppe, S. H. Oliveau, and O. J. Sovers: 1992, "Earth Rotation Parameters from DSN VLBI: 1992", IERS Technical Note 11, P. Charlot (ed.), Observatoire de Paris, 17-26.

J. M. Wahr: 1981, "The Forced Nutations of an Elliptical, Rotating, Elastic and Oceanless Earth", *Geophys. J. R. Astr. Soc.* **64**, 705-727.

J. G. Williams, X X Newhall, and J. O. Dickey: 1991, "Luni-Solar Precession: Determination from Lunar Laser Ranges", *Astron. Astrophys.* **241**, L9-L12.

J. G. Williams and W. G. Melbourne: 1982, "Comments on the Effect of Adopting New Precession and Equinox Corrections", *High-Precision Earth Rotation and Earth-Moon Dynamics*, 293-303, O. Calame ed., D. Reidel Publishing Co.

J. G. Williams and E. M. Standish: 1989, "Dynamical Reference Frames in the Planetary and Earth-Moon Systems", *Reference Frames in Astronomy and Geophysics*, 67-90, J. Kovalevsky et al. editors, Kluwer Academic Publishers, Boston.

S. Y. Zhu and I. I. Mueller: 1983, "Effects of Adopting New Precession, Nutation and Equinox Corrections on the Terrestrial Reference Frame", *Bulletin G od sique* **57**, 29-42.

Table 1  
Nutation terms estimated in joint VLBI/LLR data reduction  
(corrections the IAU (1980) theory)

Period (days)	Obliquity	Longitude
Linear		-3.79 4:0.09 mas/yr
6798.4 in-phase	2.124:0.08 mas	-10.87 ± 0.46 mas
out-of-phase	2,984:0.06	6.04 ± 0.32
3399.2 in-phase	-0.29 ± 0.06	2.51 ± 0.21 -
out-of-phase	-0.35 4:0.05	-1.21 ± 0.17
429.8 in-phase	0.09 ± 0.03	---0.04 ± 0.07
out - of-phase	-0.18 ± 0.03	-0.45 ± 0.08
365.3 in-phase	2.103:0.03	4.733:0.07
cmt-of-phase	-0.50 ± 0.03	1.373 0.07
1 S2.6 in-phase	-0.64 ± 0.02	1.58 ± 0.05
cmt-c)f-phase	-0.55 ± 0.02	-1.30 ± 0.06
13.7 in-phase	0.04 ± 0.03	0.00 ± 0.05
out -of-phase	0.07 ± 0.02	0.00 ± 0.07

UT1 tidal terms estimated in joint VLBI/LLR data reduction

Tidal mode	cos	sin
S2	-3.5 ± 1.3 $\mu$ s	19.53: 1.3 $\mu$ s
M2	-18.7 ± 1.5	21.2 ± 1.5
N2	-12.3 ± 1.1 -	-0.5 ± 1.1
K1	20.43: 1.9	---2.3 ± 1.9
P1	-4.3 ± 1.1	-10.1 ± 1.1
O1	-23.1 ± 3.6 -	-46.4 ± 3.5

VLBI and LLR polar motion and UT1-UTC rate offsets from a priori series

	X (mas/yr)	Y (mas/yr)	UT1-UTC (ms/yr)
VLBI	0.06 ± 0.04	0.264:0.04	-0.005 ± 0.003
LLR	-0.08 ± 0.04	0.55 ± 0.05	0.005 ± 0.003

Table 2  
Cartesian coordinates of DSN radiotelescopes at epoch 1988.0  
in the VLBI terrestrial frame (geocentric metric)

Site	Antenna	X (m)	Y (m)	Z (m)
Goldstone	DSS 12	-2350443.690	-4651980.828	3665630.964
	1)ss 13	-2351129.072	-4655477.097	3660956.960
	DSS 14	-2353621.129	-4641341.552	3677052.349
	DSS 15	-2353538.676	-4641649.511	3676670.038
Tidbinbilla	DSS 42	-4460980.821	2682413.543	--3674582.264
	DSS 43	-4460894.396	2682361.587	-3674748.760
	DSS 45	-4460935.049	2682765.724	-3674381.591
Madrid	DSS 61	4849245.252	-360278.293	4114884.383
	DSS 63	4849092.685	-360180.698	4115109.050
	DSS 65	4849336.780	-360488.992	4114748.714

Table 3  
Cartesian coordinates of LLR tracking stations at epoch 1988.0  
in the LLR terrestrial frame (geocentric metric)

Station	X (m)	Y (m)	Z (m)
McDonald*	-1330782.265	-5328755.372	3235697.699
Haleakala	-5466007.471	-2404427.063	2242188.555
Grasse	4581692.399	556194.929	4389354.954

\* The coordinates given here are those of the 2.7 m telescope. The positions of the three McDonald LLR stations (the 2.7 m telescope, the original McDonald Laser Ranging Station, and the new McDonald Laser Ranging Station) were constrained together in the analysis, based on ground ties.

Table 4  
 Cartesian coordinates of DSN radiotelescopes and LLR tracking stations  
 at epoch 1988.0 in the unified ITRF92 frame

Station	X (m)	Y (m)	Z (m)
DSS 12	-2350443.676	-4651980.838	3665630.978
DSS 13	-2351129.045	-4655477.084	3660956.933
DSS 14	-2353621.115	-4641341.540	3677052.355
DSS 15	-2353538.673	-4641649.540	3676670.049
DSS 42	-4460980.822	2682413.518	-3674582.264
DSS 43	-4460894.394	2682361.548	-3674748.775
DSS 45	-4460935.051	2682765.700	-3674381.594
DSS 61	4849245.258	-360278.299	4114884.408
DSS 63	4849092.680	-360180.692	4115109.057
DSS 65	4849336.787	-360488.958	4114748.700
McDonald	-1330781.245	-5328755.584	3235697.713
Haleakala	-5466006.954	-2404428.192	2242188.537
Grasse	4581692.355	556195.826	4389354.975

Table 5  
 Transformations from the unified ITRF92 frame  
 to the VLBI and LLR frames

VLBI			LLR		
$T_1'$	-0.43	0.5 cm	$T_1''$	-5.3	$\pm 6.4$ cm
$T_2'$	$0.8 \pm 0.5$	cm	$T_2''$	$3.0 \pm 8.3$	cm
$T_3'$	$0.04 \pm 0.5$	cm	$T_3''$	$1.1 \pm 6.5$	cm
$R_1'$	$-0.3 \pm 0.2$	mas	$R_1''$	$-1.7 \pm 4.2$	mas
$R_2'$	$0.24 \pm 0.2$	mas	$R_2''$	$-1.2 \pm 2.4$	mas
$R_3'$	$0.6 \pm 0.2$	mas	$R_3''$	$40.0 \pm 1.9$	mas
$D'$	$0.0 \pm 0.1$	$10^{-8}$	$D''$	$-0.1 \pm 0.8$	$10^{-8}$

Table 6  
Comparison of VLBI/LLR frame tie with observed source position differences  
between the IERS celestial frame and the frame of planetary ephemeris DE200

Source	measurement	direction"	Observed ICRF-DE200 (mas)	VLBI/LLR ICRF-DE200 (mas)
0202 + 149	Vega 1 & 2	$\hat{\alpha}$	$14 \pm 12$	$5 \pm 3$
0202 + 149	Vega 1 & 2	$\hat{\delta}$	$-15 \pm 12$	$-9 \pm 4$
0250 + 178	Phobos II	$\cos 8^\circ \hat{\alpha} + \sin 8^\circ \hat{\delta}$	$1 \pm 6$	$3 \pm 3$
0423 + 233	Phobos 11	$\cos 42^\circ 6' + \sin 42^\circ \hat{\delta}$	$0 \pm 3$	$-1 \pm 3$
1937 + 210	pulsar timing	$\hat{\alpha}$	$3 \pm 5$	$11 \pm 3$
1937 + 210	pulsar timing	$\hat{\delta}$	$-11 \pm 12$	$-7 \pm 4$

\* Position differences are given in plane-of-sky coordinate differences in the direction of increasing right ascension ( $\hat{\alpha}$ ) and increasing declination ( $\hat{\delta}$ ).